9/4/25

Lecture 04

$\neg(x^2 > 0) \equiv x^2 \leq 0$

$x \in \mathbb{Z}^+ \cup \{0\}$

$\forall x \forall y \;\; y > x \equiv F$ counterexample : $x = 2$, $y = 0$

Think of $\forall x \forall y \;\; y > x$ as :

y

x	0	1	2	3
1		(1,1)	(1,2)	(1,3)
2		⋮	⋮	⋮
3				

· Check if <u>every</u> combination is true

$\forall x \exists y \;\; (y > x) \equiv T$

$\forall x \exists y \;\; (y > x) \equiv$ "for every x is there at least one y where $y > x$"

$\exists y \forall x \;\; (y > x) \equiv$ " is there one y that is greater than <u>all possible</u> x"

$\exists x \forall y \exists z \;\; x + y > z \equiv T$

$\forall x \forall y \exists z \;\; x + y > z \equiv F$ counter example: x selects 0
y selects 0

$M(x,y)$ = x sent an email to y

- Logically express "everyone else"

  $\forall x \forall y ((x \neq y) \rightarrow M(x,y))$

- Logically express "someone else"

  $\forall x \exists y ((x \neq y) \wedge M(x,y))$

- Logically express "exactly one"

  $\exists x (L(x) \wedge \forall y ((x \neq y) \rightarrow \neg L(y)))$

- Logically express "at least 2"

$\exists x \exists y (x \neq y \wedge M(\text{Taeho}, x) \wedge M(\text{Taeho}, y))$

- Logically express "exactly 2"

$\exists x \exists y [x \neq y \wedge M(\text{Taeho}, x) \wedge M(\text{Taeho}, y) \wedge \forall z (z \neq x \wedge z \neq y \rightarrow \neg M(\text{Taeho}, z)]$

Argument

h_1
h_2
h_3

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c

$(h_1 \wedge h_2 \wedge h_3) \rightarrow c$